ECNS 204

Principles of Microeconomics

Chapter 7 (Law of Diminishing Returns) – Silberberg and Ellis

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Inputs and Outputs; Marginal Product

- In Chapter 7, we take a more in-depth approach to studying the production process
- Production takes place when various *inputs* are combined to produce some sort of *output*.
- We refer to these inputs as the *factors of production*.
- Typically, we express the production process mathematically with a production function, as follows:

$$y = f(K, L)$$

where y is output, K is capital, and L is labor.

- From the production process, we distinguish between:

marginal product of a factor input

vs.

total product

Consider the following table:

Q_{labor}	Total Product (TP)	Marginal Product (MP)	Average Product (AP)
1	3	3	3
2	8	5	4
3	15	7	5
4	21	6	5.25
5	26	5	5.2
6	30	4	5
7	33	3	4.71
8	35	2	4.38
9	36	1	4
10	36	0	3.6
11	35	-1	3.18

- We see that we can write both MP and AP in terms of the TP:

MP = TP(n) – TP(n-1)

-That is, marginal product is equal to the total product of n laborers minus the total product of (n-1) laborers.

AP = TP(n)/n

-That is, the average product is equal to the total product of n laborers divided by the total number of laborers

- We also see that, after some point, the law of diminishing marginal product holds:

"As one input of production is added to a fixed amount of other inputs, after some point, the marginal product of the variable input continually diminishes"

- Why is the above quote accurate? Perhaps it is best shown by considering the consequences of the contrary law:

Ex. If workers had increasing marginal product, with a constant wage rate, what does this imply about firm size? It implies that it would be infinite.

- The law of diminishing marginal returns certainly does not rule out increasing MP over some initial level of factor inputs, as is shown in the table on the previous slide.
  - For example, consider a boulder that cannot be moved by one person.

- Why might MP actually be negative at some point? Can you think of an example?

Profit maximization and factor demand

- The firm cares about the return from hiring an additional work…we call this the *value of the marginal product*.
- With our previous example, if the price of output is $10/unit, then we would have the following values:

Q_{labor}	Value of Total Product (VTP)	Value of Marginal Product (VMP)	Value of Average Product (VAP)
1	$30	$30	$30
2	80	50	40
3	150	70	50
4	210	60	52.5
5	260	50	52
6	300	40	50
7	330	30	47.1
8	350	20	43.8
9	360	10	40
10	360	0	36
11	350	-10	31.8

- Now, suppose that the competitive market wage is $40/day. With this information, we can add two more columns to our previous table: total labor cost and rents (aka profits).

Q_{labor}	Value of Total Product (VTP)	Value of Marginal Product (VMP)	Value of Average Product (VAP)	Total Labor Costs	Rents (= VTP – TC)
1	$30	$30	$30	$40	-$10
2	80	50	40	80	0
3	150	70	50	120	30
4	210	60	52.5	160	50
5	260	50	52	200	60
6	300	40	50	240	60
7	330	30	47.1	280	50
8	350	20	43.8	320	30
9	360	10	40	360	0
10	360	0	36	400	-40
11	350	-10	31.8	440	-90

As we can see, profits are maximized at the point where the VMP = wage.

-As long as the VMP > w, then it is in the firm's best interest to hire one more worker.

<u>Law of factor demand</u>

If the wage of any factor decreases, the firm will choose to hire additional units (a vice versa), all else being equal.

Behavior of Firms under Common Property Ownership

- Here, we are going to consider an alternative to profit maximizing behavior.

- Specifically, we will consider a common property regime, where no person has the right to exclude others from a parcel of land.
  - For example, take deep-sea fishing
    - On high seas, no property rights are established in either the fish or other ocean resources
    - Beyond each country's territorial and offshore limits, there is, in general, no law and no legal protection of a natural resource (i.e., anyone who can catch fish on open seas can keep those fish without paying fees to an owner)

- Below, we will use an example of common property vs. private property (with a profit maximizing owner) to illustrate differences in production outcomes between the two different property rights regimes.

Assumptions of common property ownership

- No one owns resource (e.g., plot of land)
- No one person can exclude another from using the land
- Communal sharing in the output occurs
  - Land is managed by a commune and all members share equally in the output of the land
  - Anyone who wishes to join the commune may
  - Instead of wages, each worker shares equally in the value of the total product

Assumptions of private property ownership

- Owned by a profit maximizing farmer
- Farmer hires workers and workers are paid the prevailing competitive wage

Example problem

- Suppose the output of wheat on a given farm is given by the following production function, where Q_{labor} represents the labor input (e.g., number of workers):

Q_{labor}	Total Product (TP)	Marginal Product (MP)	Average Product (AP)
1	5	5	5
2	13	8	6.5
3	21	8	7
4	27	6	6.75
5	32	5	6.4
6	34	2	5.67
7	35	1	5
8 or more	35	0	4.375

- Now, assume that the market price of wheat is p = $10/bushel
- Further, assume all laborers can make $50 competitive wage per day in their next best alternative
- With this information we can modify the table on the previous slide such that the TP, MP, and AP represent the VTP, VMP, and VAP, respectively. We can also add a column indicating the marginal cost of labor and a column indicating the total cost of labor. Finally, based on these values, we can add a last column that shows total profits (i.e., VTP minus TC.

Q_{labor}	**Value of Total Product (VTP)**	**Value of Marginal Product (VMP)**	**Value of Average Product (VAP)**	**Marginal Cost of Labor (MC)**	**Total Cost of Labor (TC)**	**Total Profits**
1	50	50	50	50	50	0
2	130	80	65	50	100	30
3	210	80	70	50	150	60
4	270	60	67.5	50	200	70
5	320	50	64	50	250	70
6	340	20	56.7	50	300	40
7	350	10	50	50	350	0
8 or more	350	0	43.75	50	400	-50

- a.) Suppose the land is held as "common property." That is, no one owns it and anyone who wishes can work on the farm and share equally in the output. Given the values in the table on the previous slide, how many people will farm in this case? Is the outcome efficient? Why or why not?
  - Ans. Under common property, people will join and work the farm up to the point where their next best alternative (i.e., the competitive market wage) is equal to the value of the average product.
  - In this case, referencing the values in the table, we see that 7 people will join the commune and work the farm.
  - Let's come back to the efficiency questions after we consider the private property outcome…

- b.) Suppose the land is privately held by a profit maximizing owner. How many workers would the owner hire? Is it an efficient allocation of resources?
  - Ans. A profit maximizing owner will hire up to the point where w = VMP. So, 5 workers are hired.

- Back to efficiency…it turns out that the common property outcome is economically inefficient and the private property outcome is efficient. Why?
  - The key is to recognize what the competitive market wage represents…that is, it represents what the workers could be ***producing*** elsewhere.
  - In common property, workers continue to join and work the farm past the point at which their VMP is less than or equal to what they could be producing elsewhere.
    - In our example, workers 6 and 7 could be reallocated to a competitive labor market to be producing more output for society (i.e., ***they could be producing more elsewhere***)
  - In private property, profits are maximized, but this is not necessarily why the outcome is socially efficient (e.g., monopolists maximize profits, but they are not efficient producers from a social perspective). The private property outcome is efficient because there is no possible reallocation of workers such that more output could be produced.
  - Also, see that in common property "rents are dissipated" away entirely. That is, $350 worth of farm output is produced, but $350 worth of nonfarm output is forgone…no net social gains have occurred.

- Q. If we were to apply the above logic to a natural resource such as a fish population, what might be a consequence of overproduction?

More on Diminishing Marginal Products

- To this point, we have considered firms or producers that have available only one variable input into the production process (i.e., labor)
- Let's expand upon this and consider a firm with two variable inputs: labor (L) and capital (K)
- We know that a profit maximizing firm will hire inputs up to the point where:

$$VMP_L = w_L$$

and

$$VMP_K = w_K$$ (w_K is often referred to as the "rental rate of capital")

From these equations, we can rewrite them as:

$$\frac{p*MP_L}{w_L} = 1$$

and

$$\frac{p*MP_K}{w_K} = 1$$

$$\frac{p*MP_L}{w_L} = \frac{p*MP_K}{w_K}$$

This says that in order to maximize the net benefits produced by the firm, the ratio of the value of the marginal product of each factor to its wage must be equal for all factor inputs

$$\frac{p*MP_L}{w_L} = \frac{p*MP_K}{w_K}$$

- Put differently, to maximize net benefits, the additional product forthcoming from each input per dollar spent on that input must be the same for all inputs.
  - **Thus, marginal product per dollar spent must be the same for all inputs (this is referred to as "equating the margins")**.

Example

- Suppose by spending $500 on additional labor, a firm could produce two new computers.
- But, if the firm instead used that $500 to purchase new equipment, it could produce three new computers.
- Here, the firm is not allocating labor and capital to its fullest potential
  - If the firm spends $500 less on labor and $500 more on capital, there will be a net increase of one computer
  - This kind of reallocation of inputs is possible anytime the equation shown above is violated.

- Another example of "**equating the margins**": https://montana.techsmithrelay.com/wwVw