

Exam #1 Review Questions (answers)
ECNS 303
October 4, 2011

1.) Exogenous vs. Endogenous variables

Consider a potential criminal. The individual maximizes his or her utility by choosing how much labor to supply towards legitimate work opportunities (L_w) and how much labor to supply towards crime (L_c). In doing so, this person can earn a wage w in the legal labor market and a return to crime n in the illegal labor market. In the illegal labor market, the potential criminal faces costs of punishment (e.g. time in jail) C with a positive probability of apprehension P . Of course, this individual must maximize subject to a time and budget constraint (but let's not worry about that for now). Categorize the variables listed above as either endogenous or exogenous to our utility maximization model.

Exogenous variables: w, n, C, P (These are all determined outside of the model)

Endogenous variables: L_w, L_c (These are "choice" variables that are all determined by the utility maximizer)

2.) GDP

Consider a country that produces only two products: computers and automobiles. Sales and price data for these two products for two different years are as follows:

Year	#computers sold	Price per computer	# automobiles sold	Price per automobile
2000	500,000	\$6,000	1,000,000	\$12,000
2010	5,000,000	\$2,000	1,500,000	\$20,000

a.) Calculate nominal GDP in 2000 and 2010.

Nominal GDP in 2000 = $(500,000)(\$6,000) + (1,000,000)(\$12,000) = \$15 \text{ billion}$

Nominal GDP in 2010 = \$40 billion

b.) Calculate real GDP in 2010 using 2000 as the base year.

Real GDP in 2010 = $(5,000,000)(\$6,000) + (1,500,000)(\$12,000) = \$48 \text{ billion}$

c.) Calculate the GDP deflator in 2010.

GDP deflator in 2010 = $\text{Nominal GDP in 2010} / \text{Real GDP in 2010} = \$40 \text{ billion} / \$48 \text{ billion} = 0.8333$

d.) Calculate the CPI in 2010 using 2000 as the base year.

CPI in 2010 = $(\text{price of basket of goods and services in current year}) / (\text{price of basket of goods and services in base year})$

$$= \{(\$2000 * 500,000) + (\$20,000 * 1,000,000)\} / \{(\$6,000 * 500,000) + (\$12,000 * 1,000,000)\}$$
$$= 1.4$$

3.) National Income Accounting

a.) What are the 4 components of GDP

$$Y = C + I + G + NX$$

where C is consumption, I is investment, G is gov't purchases, and NX is net exports

b.) Assume the economy is described by the following equations:

$$Y = \bar{Y} = F(\bar{K}, \bar{L}) = 1,200$$

$$Y = C + I + G$$

$$C = 125 + 0.75(Y - T)$$

$$I = I(r) = 200 - 10r$$

$$G = \bar{G} = 150$$

$$T = \bar{T} = 100$$

Given the above equations, solve for the value of savings (S).

We can rearrange the income equation to obtain

$$Y - C - G = I$$

The left-hand side of the above equation simply represents national savings and, as we recall, savings equals investment

$$S = I = Y - C - G.$$

Plugging in the given values we obtain

$$S = 1200 - (125 + 0.75(1200 - 100)) - 150 = 100$$

Since savings equals investment, we can solve for r

$$100 = 200 - 10r$$

$$\Rightarrow r = 10$$

c.) Manipulate the savings equation to separate the saving of the private sector from that of government.

To do this, we add and subtract T from the right-hand-side of the savings equation

$$S = (Y - T - C) + (T - G)$$

where the first term is private savings and the second term is public savings.

4.) Cobb-Douglas Production Function

Consider the Cobb-Douglas production function

$$F(K, L) = AK^\alpha L^{(1-\alpha)}$$

Show that the MPL is proportional to output per worker.

First, we calculate the MPL by taking the partial derivative with respect to L .

Next, we rearrange so as to solve for the right-hand-side in terms of Y/L

$$\begin{aligned} MPL &= \frac{\partial F(K, L)}{\partial L} = (1 - \alpha)AK^\alpha L^{-\alpha} = \frac{(1 - \alpha)AK^\alpha L^{1-\alpha}}{L} \\ &= (1 - \alpha) \frac{Y}{L} \end{aligned}$$

5.) Solow Growth Model

Consider an economy described by the production function:

$$Y = F(K, L) = K^{0.3} L^{0.7}$$

a.) What is the per-worker production function?

Divide through by L

$$Y/L = K^{0.3} L^{-0.3} = (K/L)^{0.3}$$

$$\Rightarrow y = k^{0.3}$$

b.) Assuming no population growth or technological progress, find the steady-state capital stock per worker, output per worker, and consumption per worker as a function of the saving rate and the depreciation rate.

Recalling that $\Delta k = sf(k) - \delta k$ and that in the steady-state $\Delta k = sf(k^*) - \delta k^* = 0$, we can write the following

$$s(k^*)^{0.3} = \delta k^*$$

$$\Rightarrow (k^*)^{0.7} = s/\delta$$

$$\Rightarrow k^* = (s/\delta)^{1/0.7}$$

From this, we can solve for ss output per worker

$$y^* = [(s/\delta)^{1/0.7}]^{0.3} = (s/\delta)^{0.3/0.7}$$

And, lastly, we can solve for consumption per worker. We know that consumption is the amount of output that is not invested. Since investment in the steady-state equals depreciation, it follows that

$$c^* = f(k^*) - \delta k^* = (s/\delta)^{0.3/0.7} - \delta (s/\delta)^{1/0.7}$$

6.) Golden Rule Steady State

Suppose the following national income accounts identity describes the economy

$$y = c + i$$

a.) Solve for the condition that describes the Golden Rule.

We know that the Golden Rule steady-state is the particular steady-state where consumption is maximized. We also know that our steady-state level of consumption is described by the following modification of our national income equation

$$c^* = f(k^*) - \delta k^*$$

To solve for the level of c^* that maximizes the above equation, we take the derivative of c^* with respect to k^*

$$dc^*/dk^* = df(k^*)/dk^* - \delta = 0$$

$$\Rightarrow MPK - \delta = 0$$

That is, at the Golden Rule level of capital, the marginal product of capital net of depreciation equals zero.

b.) Consider the decision a policymaker faces when choosing a steady-state that maximizes consumption per worker. Suppose output per worker is governed by the following production function

$$y = k^{1/2}$$

Also suppose that depreciation is 10% of capital. The policymaker must choose a savings rate s to accomplish his goal. Further suppose the policymaker must choose between the following savings rates: 0.2, 0.5, and 0.8. Which of the three savings rates maximizes consumption?

Recall again that $\Delta k = sf(k^*) - \delta k^* = 0$ in the steady-state. Which gives us the following

$$\begin{aligned} k^*/f(k^*) &= s/\delta \\ \Rightarrow k^*/(k^*)^{1/2} &= s/0.1 \\ \Rightarrow k^* &= 100s^2 \end{aligned}$$

Using this result, we can compute the steady-state capital stock for any savings rates. Let's consider each of the 3 cases:

case 1: $s = 0.2$

This implies that $k^* = 100(0.2)^2 = 4$

as a result, we know that

$$\begin{aligned} y^* &= (4)^{1/2} = 2 \text{ and} \\ \delta k^* &= (0.1)(4) = 0.4 \end{aligned}$$

Finally, this implies that $c^* = f(k^*) - \delta k^* = 2 - 0.4 = 1.6$

case 2: $s = 0.5$

This implies that $k^* = 100(0.5)^2 = 25$

as a result, we know that

$$\begin{aligned} y^* &= (25)^{1/2} = 5 \text{ and} \\ \delta k^* &= (0.1)(25) = 2.5 \end{aligned}$$

Finally, this implies that $c^* = f(k^*) - \delta k^* = 5 - 2.5 = 2.5$

case 3: $s = 0.8$

This implies that $k^* = 100(0.8)^2 = 64$

as a result, we know that

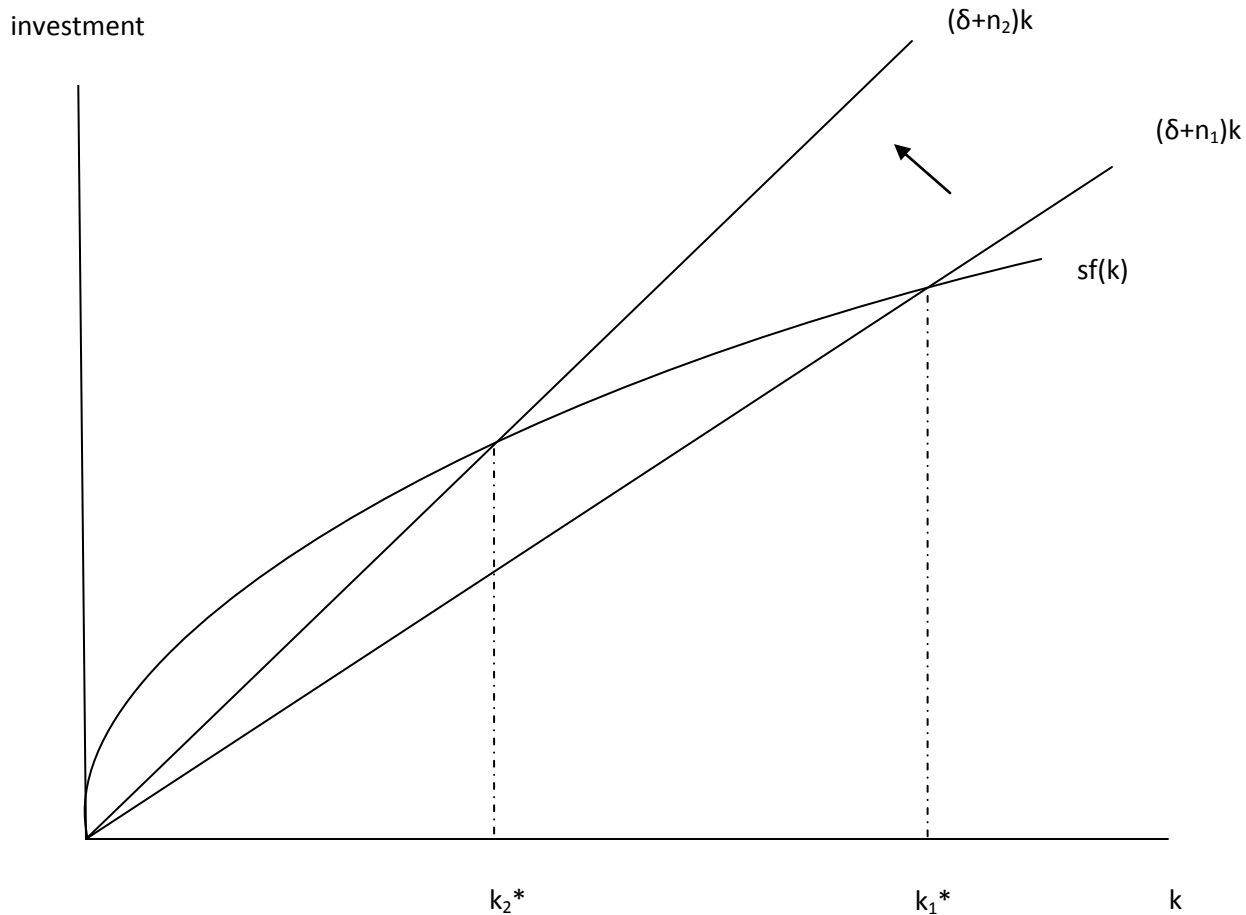
$$\begin{aligned} y^* &= (64)^{1/2} = 8 \text{ and} \\ \delta k^* &= (0.1)(64) = 6.4 \end{aligned}$$

Finally, this implies that $c^* = f(k^*) - \delta k^* = 8 - 6.4 = 1.6$

So, we see that $s = 0.5$ will be chosen as the savings rate that maximizes consumption per worker.

7.) Population Growth

Population growth gives us another explanation for why some countries are rich and others are poor. Show graphically and explain the implications of an increase in the population growth rate in the Solow model.



An increase in the rate of population growth from n_1 to n_2 leads to a reduction in the steady-state level of capital per worker. Because k^* is now lower, so is the level of output per worker. So, the Solow model predicts that countries with higher population growth will have lower levels of GDP per person.

8.) The Marginal Product of Capital

In Chapter 3 (National Income) of the textbook, the marginal product of capital was defined as the amount by which total output changes when capital rises by 1 unit, or $MPK = dY/dK$. In Chapter 8 (Economic Growth), MPK is equal to the amount by which output per effective worker rises when capital per effective worker rises by 1 unit, or $MPK = df(k)/dk$, where $k = K/EL$. Show the two definitions are the same.

In Chapter 3, $MPK = dY/dK$.

In Chapter 8, $y = Y/EL = f(k)$, where $k = K/EL$. So, $Y = ELf(k) = ELf(K/EL)$. Using the chain rule, we obtain the following

$$dY/dK = EL[df(K/EL)/dK](1/EL) = df(K/EL)/dK = df(k)/dK = MPK$$

9.) Unemployment

a.) Describe the difference between frictional and structural unemployment.

Frictional unemployment is unemployment that is caused by the time it takes workers to search for a job.

Structural unemployment is caused by wage rigidities in the labor market.

b.) What are three ways in which structural unemployment may arise?

i.) Min. wage laws

ii.) Unions/Collective bargaining

iii.) Efficiency wages

10.) Unemployment and minimum wage laws

Show with a graph how minimum wage laws can cause unemployment in the labor market.

